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Erratum

Course and Heading Changes in Significant Wind

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Nomenclature

$a_\perp$ = acceleration perpendicular to the inertial path
g = strength of gravitational acceleration
L = aerodynamic lift force
V_a = airspeed; speed of vehicle relative to air mass
V_g = ground speed; vehicle inertial speed relative to earth surface
V_w = wind speed; inertial speed of the air mass
κ = curvature of inertial path
ϕ_c = commanded bank angle (Euler roll angle)
χ = course angle (inertial track angle)
ψ = heading angle (Euler yaw orientation angle)
ψ_w = wind vector orientation

I. Ratio of Course Rate to Heading Rate of Change

IN WHAT follows, the assumption is that the vehicle is operated at constant $V_a > V_w$ in a steady wind field. Rysdyk [1] claimed

$$\frac{\dot{\chi}}{\dot{\psi}} = \frac{V_a}{V_g} \tag{1}$$

As shown below, this is not accurate. However, it is a reasonable approximation for wind conditions up to a significant percentage of the vehicle airspeed capability [2]. Expressions (6) and (7) derived in what follows imply that, in fact,

$$\frac{\dot{\chi}}{\dot{\psi}} = \frac{V_a}{V_g}\cos(\chi - \psi) \tag{2}$$

As shown below, it is of interest to note that

$$|\dot{\chi}| \lesssim \frac{V_a}{V_g}|\dot{\psi}| \tag{3}$$

II. Kinematics Approximation for the Coordinated Turn in Wind

The approximate kinematic model for coordinated turns in wind is now represented as follows:

$$\dot{x}_N = V_g \cos\chi \tag{4}$$

$$\dot{y}_E = V_g \sin\chi \tag{5}$$

$$\dot{\chi} = g V_g^{-1} \tan\phi_c \cos(\chi - \psi) \tag{6}$$

$$\dot{\psi} = g V_a^{-1} \tan\phi_c \tag{7}$$

$$\dot{V}_g = g \tan\phi_c \sin(\chi - \psi) \tag{8}$$

Equations (6) and (8) are derived from the assumption that V_a, V_w, and ψ_w remain constant, and that the "wind triangle" vector sum is represented with the following two equations:

$$V_g \cos\chi = V_a \cos\psi + V_w \cos\psi_w \tag{9}$$

$$V_g \sin\chi = V_a \sin\psi + V_w \sin\psi_w \tag{10}$$

Isolating expressions for χ and V_g and taking the derivative with respect to time leads to Eqs. (6) and (8).

III. Mechanics of Flight Perspective of the Coordinated Turn in Wind

Equation (6) can also be seen from a flight mechanics perspective as follows. Given that the aircraft is maneuvering in the horizontal plane,

$$mg = L\cos\phi \tag{11}$$

The component of the turning force perpendicular to the inertial flight path is

$$ma_\perp = L\sin\phi\cos(\chi - \psi) \tag{12}$$

The course rate of change in terms of inertial speed and curvature is

$$\dot{\chi}(t) = \kappa(t) V_g(t) \tag{13}$$

The curvature of the inertial path is related to the force perpendicular to that path as

$$a_\perp(t) = \kappa(t) V_g^2(t) \tag{14}$$

The combination of Eqs. (11–14) leads to Eq. (6).

IV. Ratio V_a/V_g as a Conservative Gain

If $0 < V_w < V_a$ then $V_g > 0$. Combining the wind triangle Eqs. (9) and (10) such that

$$2V_a V_g \cos(\chi - \psi) = V_g^2 + V_a^2 - V_w^2 \tag{15}$$

shows that then $\cos(\chi - \psi) > 0$. Combined with Expression (2) this leads to Expression (3), which states that *the magnitude of the inertial course rate is always less than that of the heading rate scaled by* $\frac{V_a}{V_g}$.

Many guidance techniques rely on feedback from a navigation system to regulate course rate of change. The typical aircraft

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responds with a roll and yaw rate to effect a heading rate of change. Therefore, the closed loop of vehicle response to desired inertial maneuvering implicitly includes the gain between course and heading rates of change. The results herein confirm that scaling a heading rate of change with $\frac{V_a}{V_g}$ is a reasonable and conservative approach to avoid interference between guidance logic and aircraft response in high wind speeds.

References

[1] Rysdyk, R., "Course and Heading Changes in Significant Wind," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 4, 2007, pp. 1168–1171.
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[2] Calise, A. J., and Preston, D., "Approximate Correction of Guidance Commands for Winds," AIAA Paper 2009-2997, 2009.